A G^n rational spline with an algebraic distance field

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WAIT 2020

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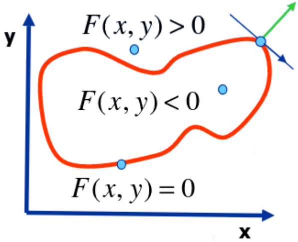
Implicit surfaces

$$f : \mathbb{R}^3 \to \mathbb{R}$$

scalar function

$$\{\mathbf{p} \in \mathbb{R}^3 : f(p) = 0\}$$

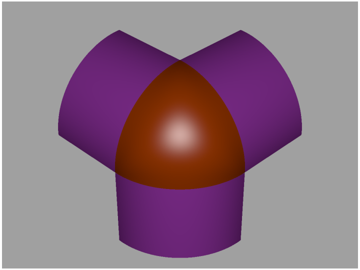
implicit surface

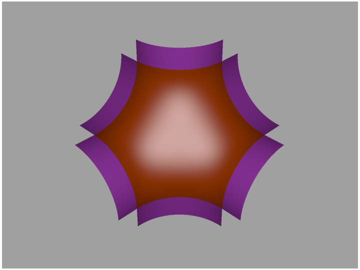


Means a space partitioning to „inside" and „outside" which define a separating surface.

I-Patch

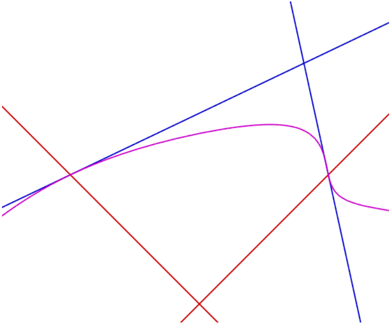
- P_i, $i = 1..n$ *implicit* primary surfaces, B_i, $i = 1..n$ *implicit* bounding surfaces
- Surfaces with the same index together represent a side of the patch
- $I = \sum_{i=1}^{n}(w_i P_i \prod_{j \neq i} B_j^{k+1}) + w \prod_{i=1}^{n} B_i^{k+1}, \; w_i, w \in \mathbb{R}$
  (kth order continuity)

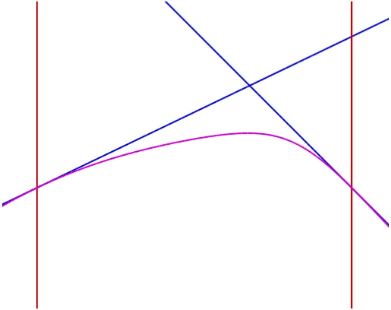




I-segment

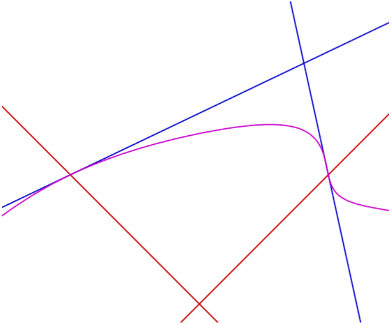
In 2D:



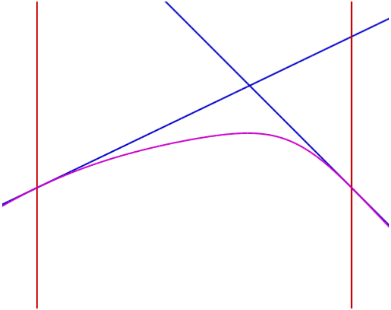


I-segment

In 2D:



General configuration



Parallel configuration

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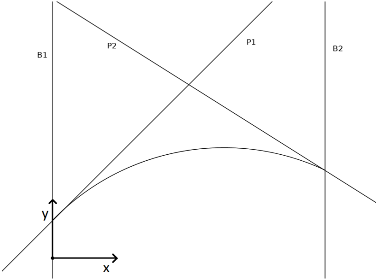
Parameterization



Use this 2D (x-y) coordinate system.

If P_1 and P_2 are graphs of functions, then the curve can be written as a function of x (rational polynomial).
Proof in paper (c.a. 2 lines)

$$y = \frac{\sum_{n=1}^{2}\left(w_n f_n(x)(x - x_{3-n})^2\right) - w_c(x - x_1)^2(x - x_2)^2}{\sum_{n=1}^{2}(w_n(x - x_{3-n})^2)}$$

Splines

What we have is:

- kth order continuity to given functions
- 3 free parameters to control shape
- Explicit rational and implicit polynomial forms (can always use the more suitable)

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We can construct spline functions from these!

- Prescribe the values and derivatives in given points and create the P_is from them

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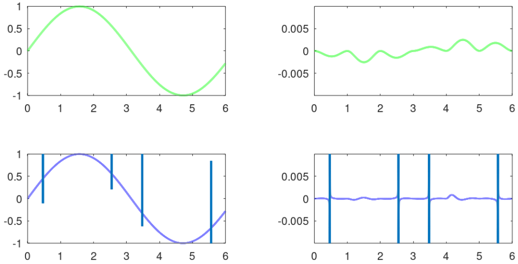
Approximation

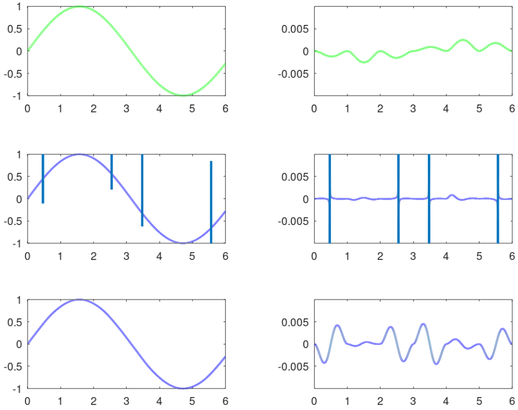


Hermite-spline (reference)

I-spline with least-squares

- Direct approximation can lead to singular results

Approximation



Hermite-spline (reference)

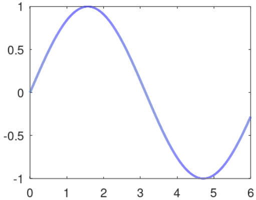
I-spline with least-squares

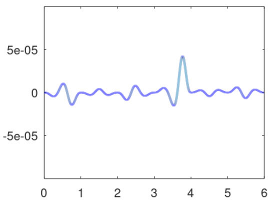
I-spline with positive coeffs

- Careful setting of coefficients is needed

Approximation

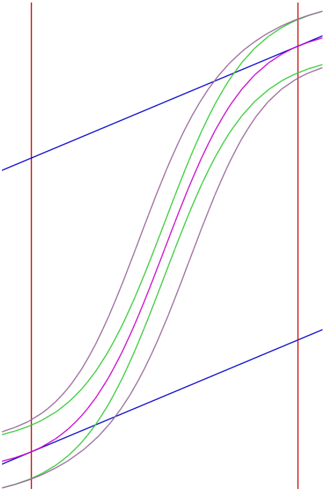
Second-order version:





Note: error scale is much lower

Distance fields



- We can also calculate algebraic distance (grey) from the curve (purple)
- More accurate than value difference (green)

Summary

- A spline basis with relatively low degree
- Can calculate algebraic error of points

Thank you for your attention!